Editors' Preface

This volume is part of Springer's book series *Outstanding Contributions to Logic*. Without doubt the contributions made by Rohit Jivanlal Parikh to logic have been deep and outstanding. Rohit is a leader in the realm of ideas, offering concepts and definitions that enrich the field and lead to new research directions.

Rohit has contributed to a variety of areas in logic, computer science and game theory: in mathematical logic they have been in recursive function theory, proof theory and non-standard analysis; in computer science, in the areas of modal, temporal and dynamic logics of programs and semantics of programs, as well as logics of knowledge; in artificial intelligence in the area of belief revision; in game theory in the formal analysis of social procedures; in all this there is a strong undercurrent of philosophy as well. With such a wide-ranging set of contributions and ever-expanding intellectual interests, we have no possibility of summarizing his work or even take the rider for an extensive tour in Parikh-land. What we do is more in the spirit of the tour brochure, listing attractions 'not to be missed'.

Finally, this volume attempts to underline the academic trajectory of a brilliant scholar whose work opened up various new avenues in research. We will briefly discuss the milestones of Parikh's scholarly work, hoping to give a sense of how he has developed his recent research program *social software* and the formal and philosophical toolkit that lies behind it. We believe that such an impressive trajectory can only help motivate the young researchers and scholars.

As our mentor, colleague, co-author, and always as our teacher, Rohit taught us a lot. This volume is a simple but honest thank you to him from all of us.

A Brisk Tour of Parikh-Land

Parikh's (1966a) paper is a classic: it introduces the notion of *semi-linearity* and proves that commutative maps of context-free languages are semi-linear sets. The paper is a revised version of a 1961 MIT report, and it was published at the invitation of Donald Knuth. Today Parikh maps and Parikh vectors are such

common usage in formal language theory that many papers use lower case as in 'parikh maps'. The paper also establishes the existence of inherently ambiguous context-free languages, which again led to a slew of results informal language theory.

Parikh (1966b) is an early paper of Parikh on *well-orderings*, a theme that would recur in his work over the next couple of decades. This paper shows that there are linear orders which are pseudo-well-ordered in that they have no recursive descending sequences, but exponentiating them yields orders that have primitive recursive descending sequences. A decade later, the paper (Parikh and de Jongh 1977) sets up a very interesting connection between hierarchies defined by closure operations and well-partial-orderings. A decade later, Parikh (1986) makes connections between this topic and two very different ones: formulas in logics of knowledge and automata theory. This paper shows precisely what levels of knowledge are achievable by a collection of communicating processes.

Parikh (1971) may well have launched the area of *bounded arithmetic*. 0 is a *small* number and whenever n is small, surely $n + 1$ is small. But a number like 10^{20} is of course not small. The paper then suggests that the length of proof establishing smallness of a number should be relevant (rather than unconstrained induction), and we can have an arithmetic that is conservative for proofs of low complexity. The paper also shows that there are formulae whose proofs are long, but the proof that they are provable could be short. Today, the study of systems of bounded arithmetic and their connection to complexity theory is a fertile area of research.

Parikh (1982, 1994) on *vagueness* are what we might call a typical Parikh phenomenon, addressing a problem of intense philosophical interest and offering conceptual guidance to those addressing it mathematically. Even more than the results, what the paper offers is clarity on concepts, and has influenced a generation of research on approximate and inexact reasoning.

The paper (Parikh 1978) and its companion (Kozen and Parikh 1981) provide an essential element that is now founded in the toolkit of every graduate student in logics of computation today. Following Kozen and Parikh, one proves a completeness theorem for the propositional dynamic logic of regular programs and combines this with a small model property leading to a decision procedure. This is perhaps the most authoritative handling of Kleene iteration in logic yet. Parikh went on to made extensive mathematical contributions to the study of dynamic and process logics.

Parikh (1983) is another landmark, a brilliant logicisation of two-person games that offered technical questions that remain open to this day. If logic can be given its meaning entirely in game theoretic terms, it is reasonable to ask what reasoning about existence of strategies in such games may mean. Game logics are a fertile area of study today.

Parikh (1984) connects logics of knowledge and non-monotonic reasoning in essential ways that would influence thinking in research on dynamic epistemic logics almost two decades later. In a series of papers over the next decade, Parikh has enriched the theory of knowledge in its interaction with communication and action, exploring intrinsic logical questions.

Moss and Parikh (1992) opens up another area, the study of topology via logic. The use of epistemic logic in this context enriches and expands our understanding of topological spaces and this study flourishes to date.

Parikh (1995) formally launches Parikh's idea of *social software*, one that has kept him creatively occupied for the last two decades, enriching interaction between logic and game theory. Social software is the analysis of social procedures by exploring their logical underpinnings. For instance, notions like social obligation, why politicians lie during election campaigns, can be studied logically.

Parikh (1999) is another landmark, this time offering a new technique, that of language splitting, in the extensively studied area of belief revision. The paper shows how we can incorporate a formal notion of relevance, which allows one's information to be split uniquely into a number of disjoint subject areas. This idea has been subsequently been extended fruitfully by Makinson and others, leading to many new directions of research.

Through all this, Parikh's contributions to fundamental logic (e.g. Parikh and Väänänen 2005) and mathematics (Parikh and Nathanson 2007) continue. His recent contributions to philosophical thought (e.g. Parikh 2013; Ginés and Parikh 2015) in the arena of logic, games, language and computation, raise a number of conceptual issues and offer new approaches which will guide research for a long time to come.

This Volume

The contributors to this volume honor Rohit Parikh and his *oeuvre* in many ways. This is not a collection of articles on one theme, or even directly connected to specific works of Parikh, but inspired and influenced by Parikh in some way, adding structures to Parikh-land, enriching it. Our goal in what follows is to say a few brief words about each of the contributions.

Juliet Floyd illustrates the transition in Parikh's interest from formal languages to natural languages, and how Parikh approached Wittgenstein's philosophy of language. In fact, the article describes, as Floyd put it why "Wittgenstein owes Parikh a big 'Thank You'."

Prashant Parikh notes that "Rohit Parikh may have been the first person to study how communication with vague expressions can be fruitful even when the speaker and addressee interpret them differently". Prashant's paper continues this line of work, introducing models from cognitive psychology to analyze vague communication.

Robert van Rooij's paper employs non-classical logic in an analysis of a well-known epistemic paradox, Fitch's Paradox. The topic is attractive for both philosophers and epistemic logicians alike, and van Rooij's contribution hopefully will initiate more interaction between these relatively separate research communities.

If we wish to take seriously Parkih's idea of social software, *money* would seem to be a central social and computational resource. Jan van Eijck and Philip Elsas orchestrate a thought-provoking Socratic dialogue on the function of money, its organizing role in society and its underlying logical principles.

Dominik Klein and Eric Pacuit focus on another important aspect of social software, and one of Parikh's recent interests: voting and political campaigns. Klein and Pacuit discuss a qualitative analysis of voters' changing opinions during a political campaign. Such analyses will be crucial to advance Parikh's program of analysis of social procedures.

Can Başkent discusses the role of classical logic in the social sofware enterprise, and offers an interesting extension of social software to paraconsistent logic, arguing that non-classical logics provide the theme with a broader domain of applications. The paper follows the footsteps of Parikh (2002) and takes a logical pluralist stand on the subject.

Epistemic logic is a central arena of Parikh's work, and much of his work is on delineating notions of knowledge, syntactically and semantically. Knowing a proposition is distinct from the sentential knowledge of the proposition. While economists in general prefer to work directly with propositions at the model level, logicians prefer to work with compositional syntax. This distinction runs through Parikh's work (Parikh 2005) and the paper by Joseph Halpern addresses the issue. He gives strong arguments why syntax can help make finer distinctions and describe notions in a model-independent manner.

Another contribution to this volume on knowledge is Johan van Benthem's exploration of epistemology. The paper is foundational, a far-reaching exploration of a number of themes weaving epistemology, dynamic logic, information, modality, and action. Many of these themes run through Parikh's oeuvre as well. (It should be noted here that Parikh was an early enthusiast for Jan Plaza's work, and that work itself is a major forerunner of dynamic epistemic logic.)

A central lesson of Parikh's work on knowledge is that communication and knowledge ought to be studied together. In their paper on *Gossip protocols*, Maduka Attamah, Hans van Ditmarsch, Davide Grossi and Wiebe van der Hoek consider agents who exchange private information about others over a telephone network. They study how many message exchanges are needed for a particular state of knowledge to be achieved.

Analyzing puzzles through logic and games is an endearing component of Parikh's style, and Sandu and Velica's paper in this volume pays tribute to it. Hintikka and Sandu (1989) introduced independence-friendly (IF) logic in order to express patterns of dependencies among quantifiers which go beyond what can be expressed in first-order logic. The paper by Gabriel Sandu and Silviu Velica offers a formulation of the Monty Hall puzzle in IF logic via a game-based modeling of the problem. In the process, they endow IF logic with a probabilistic semantics.

Amy Greenwald, Jiacui Li and Eric Sodomka establish a formally appealing connection between a specific presentation of games (called *extensive normal form games*) and a process of randomized decisions. Uniting two different approaches to

decision making provides us with a broader understanding of game theoretical decision processes.

Similarly, Jouko Väänänen discusses an iteration of a logical framework which arises from a situation of limited information. The solution is given within the context of dependence logic with team semantics.

Melvin Fitting's paper in this volume is a contribution to an important issue in modal logic, the relation between *intensions* and *extensions*. His paper proposes an appealing formal treatment of *predicate abstraction*. This paper not only discusses examples, it presents a formal proof system which is sure to be of independent interest.

Konstantinos Georgatos' paper on *epistemic conditionals* addresses belief revision, an important topic in artificial intelligence and related areas of philosophical logic. Yet another connection to Parikh's work comes in this paper's use of the logic of *subset spaces*, a topic founded by Parikh and Moss.

Dexter Kozen's contribution to this volume is a contribution to *Kleene algebra*, an area pioneered by Kozen himself. Kleene algebras may be thought of as algebraic structures, which generalize and illuminate the algebra of regular expressions. Thus, it harks back to Parikh's interest in formal language theory and dynamic logic.

Vaughan Pratt's paper shares with Kozen's a decidedly algebraic flavor. In fact, Pratt's paper takes up several topics relating category theory and syllogistic logic. Although at first this seems an unlikely match, Pratt shows how algebraic perspectives can illuminate the technical sides of logical systems which we thought we knew well.

Noson Yanofsky discusses a deeply foundational issue in theoretical computer science: algorithms. Parikh's work on the logic of programs is carried to a more abstract level. The relation between programs and algorithms is discussed using a novel category theoretic approach.

All Aboard

We hope to have presented you with a brochure-view of Parikh-land and then given an "introductory video" on the sights and sounds that you will experience when reading the book. We now invite you to board the bus. The drivers are grateful to all the authors for their contributions, and to the Series Editor Sven Ove Hansson for giving us the opportunity for this wonderful ride.

Bath, UK   Can Başkent
Bloomington, USA   Lawrence S. Moss
Chennai, India   Ramaswamy Ramanujam